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ESTIMATION OF THE VOLTAGE CONTROLLABILITY OF DISTRIBUTUION SYSTEMS WITH LOCALLY DISTRIBUTED GENERATION SOURCES

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Abstract– *This report proposes a methodology for estimation of the effects of distributed generation (DG) on the distribution network under normal steady- state operation conditions. Taking into account the characteristics of the generation units and the characteristics of the electrical network a multiple power flow analysis is presented to be suitable for investigation of the impact of the DG on the network. The methods of V-Q sensitivity, Q-V modal analysis, power transfer efficiency, voltage profile analysis, and some other direct indices are suggested for quasi- steady state estimation of the distribution system regarding the voltage and reactive power control. On the basis of the methodology proposed the impact of distributed generation on the network can be evaluated, sensitive and predisposed to voltage profile problems points and other "weak" places of the network can be localized and distinguished and measures for optimization of the voltage control and support of the voltage profile can be determined.*

Keywords- *voltage control, distributed generation, static analysis, modal analysis, V-Q sensitivity, power transfer efficiency*

I. INTRODUCTION

Distributed generation has recently become a major issue for many distribution power systems. Because of some political, economic and environmental circumstances many modern distribution power systems are penetrated by a significant number of small local generators. The conversion of the former passive networks to an active one with distributed generation faces new challenges for the planning and operation, imposing a development and use of new voltage control strategies in the distribution networks [5,6].

An important issue of the voltage control problem is the ability of the power system to maintain admissible voltage levels in the various system busses during the large variety of different conditions at which the network operates.

The variability and the stochastic inherent of loads and the generators in the different nodes of the system originates a large number of different operating conditions which have to be examined. This multidimensionality of different network topologies, different generation and load profiles in the time and space makes the problem very complex. Thus the analytical approach is recognized to be favorable for estimation of the new, in many cases forecasted operation modes.

Besides the traditional power- flow based voltage profile analysis some additional indicators are presented and used in order to provide a more extensive research of the operation mode.

II. METHODOLOGY

In order to investigate the impact of the distributed generation on the steady- state voltage profile and to localize the so called "weak" (critical) places in the network at which potential voltage control problems can arise an analytical, power- flow based approach is proposed. This approach is structural and can be easily further extended and also used in combination with other real time measurement- based approaches. The methodology described is also developed, realized and used in the STATUS application for power system analysis.

The critical data needed for the analytical power system state estimation are the data for the topology of the network and the data for the active and reactive loads and generations in the system nodes at the given estimated operational state.

The steady- state behavior of the system can be defined using the nonlinear power flow equations [1,2,3]:

$$P_k = \sum_{m \in k} V_k V_m (G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}); \tag{1}$$

$$Q_k = \sum_{m \in k} V_k V_m (G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km}), \tag{2}$$

where

P_k is the active power at node (bus) k;

Q_k - reactive power at node m;

V_k - voltage magnitude at node k;

V_m - voltage magnitude at node m,

θ_k - voltage angle at node k;

θ_m - voltage angle at node m,

and $\theta_{km} = \theta_k - \theta_m$.

The nonlinear power flow equations can be linearised around a certain operation point in Taylor series neglecting the terms after the firs two for example using the Newton- Raphson method or some of its modifications. Thus for a given network with *n* nodes the incremental equations can be written in the following matrix form:

$$\begin{bmatrix} \begin{bmatrix} \Delta P_1 \\ \Delta P_2 \\ \dots \\ \Delta P_n \end{bmatrix} \\ \begin{bmatrix} \Delta Q_1 \\ \Delta Q_2 \\ \dots \\ \Delta Q_n \end{bmatrix} \end{bmatrix} = \underbrace{\begin{bmatrix} \begin{bmatrix} \frac{\partial P_1}{\partial \theta_1} & \frac{\partial P_1}{\partial \theta_2} & \dots & \frac{\partial P_1}{\partial \theta_n} \\ \frac{\partial P_2}{\partial \theta_1} & \frac{\partial P_2}{\partial \theta_2} & \dots & \frac{\partial P_2}{\partial \theta_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial P_n}{\partial \theta_1} & \frac{\partial P_n}{\partial \theta_2} & \dots & \frac{\partial P_n}{\partial \theta_n} \end{bmatrix} & \begin{bmatrix} \frac{\partial P_1}{\partial V_1} & \frac{\partial P_1}{\partial V_2} & \dots & \frac{\partial P_1}{\partial V_n} \\ \frac{\partial P_2}{\partial V_1} & \frac{\partial P_2}{\partial V_2} & \dots & \frac{\partial P_2}{\partial V_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial P_n}{\partial V_1} & \frac{\partial P_n}{\partial V_2} & \dots & \frac{\partial P_n}{\partial V_n} \end{bmatrix} \\ \begin{bmatrix} \frac{\partial Q_1}{\partial \theta_1} & \frac{\partial Q_1}{\partial \theta_2} & \dots & \frac{\partial Q_1}{\partial \theta_n} \\ \frac{\partial Q_2}{\partial \theta_1} & \frac{\partial Q_2}{\partial \theta_2} & \dots & \frac{\partial Q_2}{\partial \theta_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial Q_n}{\partial \theta_1} & \frac{\partial Q_n}{\partial \theta_2} & \dots & \frac{\partial Q_n}{\partial \theta_n} \end{bmatrix} & \begin{bmatrix} \frac{\partial Q_1}{\partial V_1} & \frac{\partial Q_1}{\partial V_2} & \dots & \frac{\partial Q_1}{\partial V_n} \\ \frac{\partial Q_2}{\partial V_1} & \frac{\partial Q_2}{\partial V_2} & \dots & \frac{\partial Q_2}{\partial V_n} \\ \dots & \dots & \dots & \dots \\ \frac{\partial Q_n}{\partial V_1} & \frac{\partial Q_n}{\partial V_2} & \dots & \frac{\partial Q_n}{\partial V_n} \end{bmatrix} \end{bmatrix}}_{\mathbf{J}} \cdot \begin{bmatrix} \begin{bmatrix} \Delta\theta_1 \\ \Delta\theta_2 \\ \dots \\ \Delta\theta_n \end{bmatrix} \\ \begin{bmatrix} \Delta V_1 \\ \Delta V_2 \\ \dots \\ \Delta V_n \end{bmatrix} \end{bmatrix}, \tag{3}$$

Or shortly:

$$\begin{bmatrix} \mathbf{\Delta P} \\ \mathbf{\Delta Q} \end{bmatrix} = \underbrace{\begin{bmatrix} \frac{\partial \mathbf{P}}{\partial \boldsymbol{\theta}} & \frac{\partial \mathbf{P}}{\partial \mathbf{V}} \\ \frac{\partial \mathbf{Q}}{\partial \boldsymbol{\theta}} & \frac{\partial \mathbf{Q}}{\partial \mathbf{V}} \end{bmatrix}}_{\mathbf{J}} \cdot \begin{bmatrix} \mathbf{\Delta \boldsymbol{\theta}} \\ \mathbf{\Delta V} \end{bmatrix}, \tag{4}$$

where

$\mathbf{\Delta P}$ is the incremental change in bus real power,

$\mathbf{\Delta Q}$ - incremental change in bus reactive power,

$\mathbf{\Delta V}$ - incremental change in bus voltage magnitude,

$\mathbf{\Delta\theta}$ incremental change in bus voltage angle,
$\mathbf{J}$ - Jacobian matrix.

The terms of the Jacobian matrix for k and m are as follows [3]:
for $k \neq m$:

$$\frac{\partial P_k}{\partial \theta_m} = V_k V_m (G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km}); \tag{5}$$

$$\frac{\partial P_k}{\partial V_m} = V_k (G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}); \tag{6}$$

$$\frac{\partial Q_k}{\partial \theta_m} = -V_k V_m (G_{km} \cos\theta_{km} + B_{km} \sin\theta_{km}) = -V_m \frac{\partial P_k}{\partial V_m}; \tag{7}$$

$$\frac{\partial Q_k}{\partial V_m} = V_k (G_{km} \sin\theta_{km} - B_{km} \cos\theta_{km}) = \frac{\left(\frac{\partial P_k}{\partial \theta_m}\right)}{V_m}; \tag{8}$$

and respectively for $k \equiv m$:

$$\frac{\partial P_k}{\partial \theta_k} = -Q_k - B_{kk} V_k^2; \tag{9}$$

$$\frac{\partial P_k}{\partial V_k} = \frac{P_k}{V_k} + G_{kk} V_k; \tag{10}$$

$$\frac{\partial Q_k}{\partial \theta_k} = P_k - G_{kk} V_k^2; \tag{11}$$

$$\frac{\partial Q_k}{\partial V_k} = \frac{Q_k}{V_k} - B_{kk} V_k. \tag{12}$$

Under certain conditions the Jacobian matrix written above shows some specific characteristics which make it appropriate for power system state estimation [3]. Although the problem is nonlinear the relations can be assumed as nearly linear around a certain operation point [1,3].

Voltage profile analysis

The voltage profile analysis is one of the oldest and widest used methods for steady- state estimation of distribution systems. The voltage profile can be obtained by solving the power flow problem for example using the conventional Newton- Raphson method [1,3]. Although the voltage profile is evident and easy- to- understand the magnitudes at the different nodes are not always an adequate system state indicator. The main reason for this is that the voltage magnitudes in the different nodes correspond to a great number of different V-P and V-Q curves with different shapes and slopes [3]. For this reason some additional approaches for steady- state estimation will be presented.

V-Q Sensitivity

The system state is influenced by the active power **P** and the reactive power **Q**. However it may be presumed that the active power **P** preserves around a given operating point and to examine only the connection (coupling) between the increments of the bus voltage **V** and the reactive power **Q**. In that case $\mathbf{\Delta P = 0}$ and therefore [1,3]:

$$\mathbf{\Delta Q = J_R \Delta V}, \tag{13}$$

where

$$\mathbf{J_R} = \mathbf{J_{QV}} - \mathbf{J_{Q\theta}}\mathbf{J^{-1}_{P\theta}}\mathbf{J_{PV}} \tag{14}$$

is the reduced Jacobian matrix [2]. Equation (14) shows that:

$$\mathbf{\Delta V} = \mathbf{J^{-1}_R}\mathbf{\Delta Q}\,. \tag{15}$$

The matrix $\mathbf{J^{-1}_R}$ is called *V-Q reduced Jacobian matrix.* Its i[th] diagonal element defines the *self (eigen) V-Q sensitivity* at the i[th] bus [1,2]. The off- diagonal elements of this matrix define the *mutual V-Q sensitivity*.

V-Q sensitivity at each bus of the network defines the slope of the *Q-V* curve for the respective bus when calculating the pseudo steady state. When sensitivity grows, the controllability decreases, reaching zero at the verge of stability at which the bus voltage is non controllable. Negative V-Q sensitivity is an indication of unstable operation [1,2].

The advantage of this method is that it gives information for the system state as a whole and precisely defines the areas, where voltage control problems may appear.

Modal Analysis

The power system state may be estimated by calculating the eigenvalues and the eigenvectors of the reduced Jacobian matrix $\mathbf{J_R}$ defined in expression (14) [1,2]. Let

$$\mathbf{J_R} = \boldsymbol{\xi\Lambda\eta}\,, \tag{16}$$

where $\boldsymbol{\xi}$ is the right eigenvector matrix of $\mathbf{J_R}$; $\boldsymbol{\eta}$ - the left eigenvector matrix of $\mathbf{J_R}$; $\boldsymbol{\Lambda}$ - diagonal eigenvalue matrix of $\mathbf{J_R}$.

From equation (5) follows that

$$\mathbf{J^{-1}_R} = \boldsymbol{\xi\Lambda^{-1}\eta}\,. \tag{17}$$

Substituting this expression in (15) gives:

$$\mathbf{\Delta V} = \boldsymbol{\xi\Lambda^{-1}\eta}\,\mathbf{\Delta Q} \tag{18}$$

or

$$\mathbf{\Delta V} = \sum_i \frac{\boldsymbol{\xi}_i \boldsymbol{\eta}_i}{\lambda_i}\mathbf{\Delta Q}\,, \tag{19}$$

where $\boldsymbol{\xi}_i$ is the i[th] column right eigenvector of $\mathbf{J_R}$; $\boldsymbol{\eta}_i$ - is the i[th] column left eigenvector of $\mathbf{J_R}$.

Each eigenvalue λ_i and the corresponding right and left eigenvectors $\boldsymbol{\xi}_i$ and $\boldsymbol{\eta}_i$ define the i[th] mode ***Q-V*** mode. Since $\boldsymbol{\xi}^{-1} = \boldsymbol{\eta}$, expression (17) can be rewritten as:

$$\boldsymbol{\eta}\mathbf{\Delta V} = \boldsymbol{\Lambda^{-1}\eta}\,\mathbf{\Delta Q} \tag{20}$$

or

$$\mathbf{v} = \boldsymbol{\Lambda^{-1}}\mathbf{q}\,, \tag{21}$$

where $\mathbf{v} = \boldsymbol{\eta}\mathbf{\Delta V}$ is the vector of modal voltage variations; $\mathbf{q} = \boldsymbol{\eta}\,\mathbf{\Delta Q}$ - vector of modal reactive power variations.

Equation (10) contains a set of uncoupled first order equations. Thus for the i[th] mode can be written:

$$\mathbf{v_i} = \frac{1}{\lambda_i}\mathbf{q_i} \tag{22}$$

since $\boldsymbol{\Lambda^{-1}}$ from (21) is a diagonal matrix, whereas $\mathbf{J^{-1}_R}$ in (15), is in general is nondiagonal.

If $\lambda_i > 0$, the i[th] modal voltage $\mathbf{v_i}$ and the i[th] modal reactive power $\mathbf{q_i}$ variations are along the same direction, indicating that the system is controllable and stable.

If $\lambda_i < 0$, the i[th] modal voltage $\mathbf{v_i}$ and the i[th] modal reactive power $\mathbf{q_i}$ variations are along opposite directions, indicating that the system is voltage uncontrollable and unstable.

The magnitude of each modal voltage variation $\mathbf{v_i}$ equals the inverse of λ_i times the magnitude of the modal reactive power variation $\mathbf{q_i}$. Thus the magnitude of λ_i determines the degree of controllability and stability of the i[th] modal voltage. The smaller the magnitude of positive λ_i, the closer the i[th] modal voltage is to being uncontrollable.

With $\lambda_i = 0$, the i[th] modal voltage collapses because any incremental change in that modal reactive power causes infinite change in the modal voltage.

The magnitude of the eigenvalues can provide a relative measure of the voltage controllability. Theoretically the eigenvalues can not provide an absolute measure because of the nonlinearity of the problem if no other additional considerations are made [1,3]. However since the voltage control problem concerns the system behaviour under normal steady state conditions, at which the relations remain quasi linear around a certain operation point the modal analysis shows an adequate performance [1,2,3]. The modal analysis can be further extended by finding the following participation factors [2]:

- Bus participation factors (BPF)

The relative participation k in mode i is given by the bus participation factor:

$$P_{ki} = \xi_{ki}\eta_{ik} . \tag{23}$$

The participation factor P_{ki} determines the contribution of λ_i to the V-Q sensitivity at bus k. For all small eigenvalues bus participation factors determine the areas and nodes prone to voltage control problems. The sum of all the bus participations for each mode is equal to unity because the right and left eigenvectors are normalized. Bus participation factors clearly distinguish the critical nodes and areas at which a voltage control problems can arise. Bus participation factor in a given mode also indicates the effectiveness of remedial actions applied at that bus in improving the mode.

- Branch participation factors (BrPF)

With the angle and voltage variations for both sending and receiving ends known, the linearised change in branch reactive loss can be calculated. The relative participation of branch *j* in mode *i* is given by the participation factor

$$P_{ji} = \frac{\Delta Q_{loss\ \ for\ branch\ j}}{\max\left|\Delta Q_{loss\ \ for\ all\ branches}\right|} . \tag{24}$$

Branch participation factors indicate, for each mode, which branches consume the most reactive power in response to an incremental change in reactive load. Branches with high participation are either weak links or are heavily loaded. Branch participation factors are useful for identifying to alleviate voltage control problems.

Like the V-Q sensitivity method, the modal analysis method gives information for the system controllability as a whole and clearly distinguishes the areas, where problems could appear. In addition, through the participation factors, the Q-V modal analysis gives information for the mechanism in which the processes takes place and the role, which the distinct elements of a EPS play in the system state.

Power transfer efficiency (PTE)

Power transfer efficiency is one of the indicators proposed and developed in the program *STATUS* [4]. An important factor for the normal operation of the system at steady-state is the ability of the power system to transfer efficiently active and reactive power from the generation area to the consumption area. Power transfer efficiency focuses on this fact.

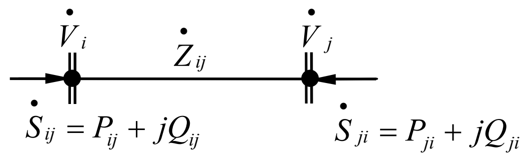

Fig.1.One line diagram for a particular branch of the power system.

For any branch of the transmission (distribution) system connecting busses *i* and *j* the power transfer efficiency (PTE) can be defined as a ratio between the received power and the sent power (Fig. 1). Since the transferred power is a complex number, real and reactive power transfer efficiencies can be defined:

$$realPTE_{ij} = \left| \frac{\min(P_{ij}, P_{ji})}{\max(P_{ij}, P_{ji})} \right| \tag{26}$$

and

$$reactivePTE_{ij} = \left| \frac{\min(Q_{ij}, Q_{ji})}{\max(Q_{ij}, Q_{ji})} \right|. \tag{27}$$

Active and reactive power transfer efficiencies show the state of the transmission system and its elements indicating which the most overloaded branches are. Branch with power transfer efficiency close to 1 presents normal operation. Branch with power transfer efficiency close to 0 presents abnormal operation of the element. Power transfer efficiencies can be defined for a particularly chosen branch, for the transmission system of a selected area, or for the transmission system of the entire power system.

III. Implementation

The methodology described here is implemented and examined in the *STATUS* program for power system analysis. A significant number of different distribution systems with different characteristics are tested in order to illustrate the performance of the methodology described. Shortly a simple representative test case for an existing 21 bus, 0,4 kV distribution network of a village near Sofia, Bulgaria will be presented (Fig. 1). More detailed description of the system and the results obtained can be found in [3, 7, 8]. All the quantities are presented in named units.

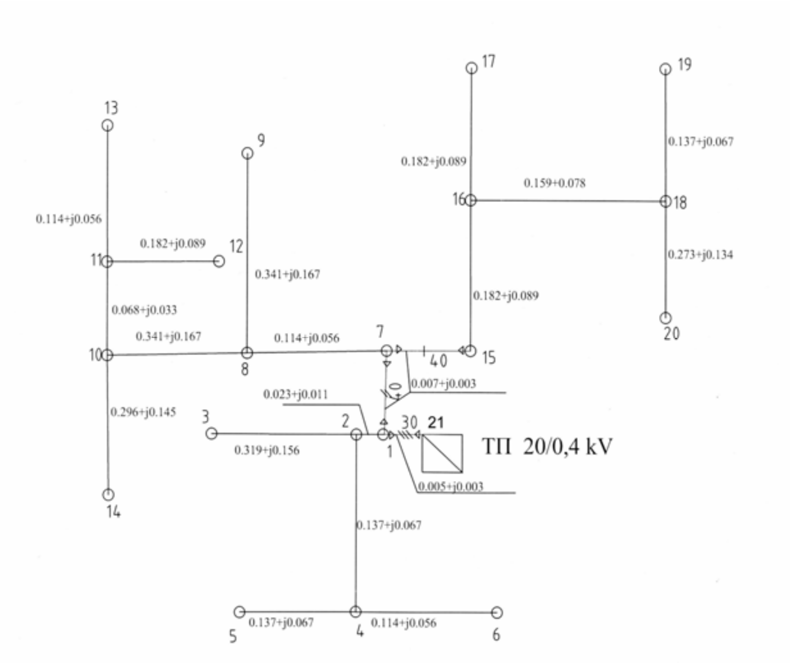

Fig 1. One- line scheme of the tested system

Voltage profile analysis

The results of the power flow solution define the voltage profile. For the particular tested system there are five buses only, which have a voltage outside the acceptable boundaries of +-5%. These are the buses 10, 11, 12, 13 and 14 (Fig. 2).

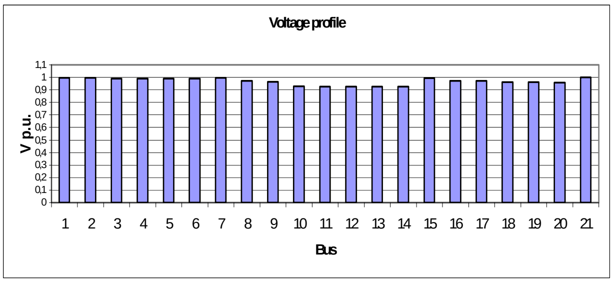

Fig 2.Voltage profile of the tested system

V-Q Sensitivity

The buses with the greatest V-Q sensitivities, are buses 10,11,12,13 and 14 (fig. 3).

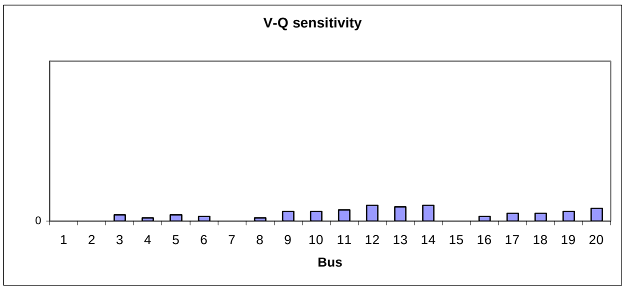

Fig 3.Voltage profile of the tested system

Bus participation factors

The size of the participation factor shows in what degree the applied measures will contribute for improving the mode. In the examined system buses 10, 11, 12, 13 and 14 differ with a great participation factor (Fig. 4).

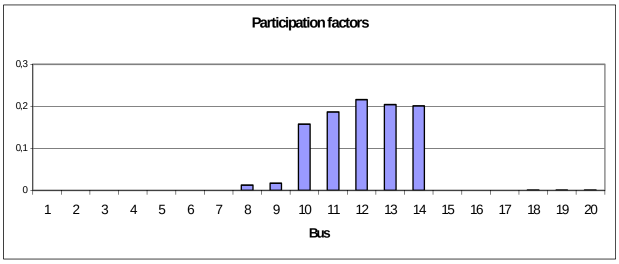

Fig 4.Voltage profile of the tested system

Branch participation factors

The participation factors of the branches in each state show which branches consume the greatest quantity of reactive energy, as a result of load increase. Branches with great participation factor are either weak or overloaded lines. For the examined network in the initial steady state the greatest participation factors have branches (7,8), (8,10) and (15,16) (Fig. 5).

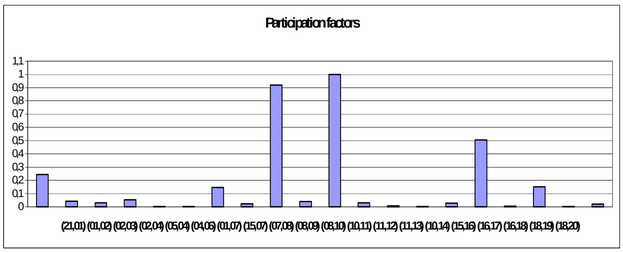

Fig 5.Branch participation factors

IV. CONCLUSION

A set of methods and indicators for power system state estimation regarding the voltage control problem are presented. The methodology presented here can be used for distinction of the critical nodes, areas and branches in the power system at which remedial measures for improving the mode of operation should be taken. The indicators and methods observe the performance of the system as a whole which make them appropriate for optimal (coordinated or local) voltage control. Additionally due to the participation factors and the indicators presented the impact of the distribution generators on the distribution network can be evidently estimated. Considering the results obtained the method of the modal analysis shows best characteristics while examining the system state.

V. ADDITIONAL REMARKS

This work was carried out in the scope of NoE DERlab (SES6-CT-2005-518299) http://www.der-lab.net.

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